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# A THERMAL STRESS SOLUTION FOR MULTILAYERED COMPOSITE CYLINDERS

MARK D. WITHERELL



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# A THERMAL STRESS SOLUTION FOR MULTILAYERED COMPOSITE CYLINDERS

Mark D. Witherell

## ABSTRACT

An elastic thermal stress solution is presented for a multilayered composite cylinder subjected to a radial temperature field. The solution assumes each layer to be at a uniform temperature, therefore, many layers are required to model regions with large gradients in temperature. Previously developed closed-form solutions for a monolayered cylinder subjected to a uniform temperature change and loadings of internal and external pressures are used to construct the multilayered solution. Results produced by the theoretical solution have been compared to those generated from a finite element analysis with excellent agreement. Finally, by combining a previously developed multilayered stress solution for loadings of internal pressure, external pressure, and axial force with the plane-strain thermal stress solution of this report, the multilayered thermal stress solution for generalized plane-strain boundary conditions is shown to be easily obtained.

## NOMENCLATURE

$a$ - inner radius of a layer
$A_{ij}$ - elements of the compliance matrix for an orthotropic material
$b$ - outer radius of a layer
$p$ - internal pressure on a layer
$q$ - external pressure on a layer
$r$ - radial position within a layer
$\alpha$ - thermal expansion coefficient
$\Delta T$ - uniform temperature change of layer
$\epsilon$ - strain
$\sigma$ - stress

Subscripts

$a,b$ - evaluated at inner and outer radial location
$r,\theta,z$ - radial, hoop, and axial directions

Superscripts

$pq$ - identifies stresses and strains that are associated with the monolayered stress solution for internal pressure 'p' and external pressure 'q'.
$T$ - identifies stresses and strains that are associated with the monolayered stress solution for thermal loading $\Delta T$.

## INTRODUCTION

In certain cylindrical pressure vessel applications, such as cannon, there exists a severe thermal environment in which the vessel is exposed to high temperatures and large heat inputs. In recent years there has been increased interest in exploiting the unique qualities of composite materials in the design of large caliber cannons. Most often, composite materials are employed as jackets in a multilayered cannon construction. In order for design and analysis of these types of vessels to be carried out, a thermal stress solution for thick-walled multilayered composite cylinders must be used. The problem of constructing the multilayered thermal stress solution involves applying the proper boundary conditions to the appropriate monolayered solutions. The two monolayered solutions used in this construction process involve constant temperature thermal loading and loading of internal and external pressures. The solution involving internal and external pressure is necessary to preserve continuity of the structure as each layer expands or contracts due to its temperature change. In both of these stress solutions, each layer is considered to be an orthotropic material and under plane-strain boundary conditions, i.e., each layer has zero axial strain. The monolayered thermal stress solution comes from previous work by Gerstle and Reuter (1975) of Sandia Laboratories. Their solution is actually a special case of that given by Tauchert (1974). An interesting and important result of this solution is that--unlike isotropic cylinders, which are stress-free for a uniform temperature change--orthotropic cylinders sustain radial, hoop, and axial stresses as a result of a uniform temperature change. The monolayered stress solution for loadings of internal and external pressures is given by Lekhnitskii (1963). Both the work of Gerstle and Reuter and that of Lekhnitskii represent closed-form solutions for the loadings and boundary conditions mentioned. The essential components of these solutions are more fully discussed below.

## GEOMETRY, MATERIAL, AND LOADING DEFINITION

A multilayered cylinder can be viewed as an assembly of many single-layered cylinders. It is appropriate, therefore, to begin with a review of the monolayered orthotropic cylinder problem for thermal loading and loadings of internal and external pressures. It should be mentioned at this point that the nomenclature used in this report has been chosen to be consistent with a previous work by Witherell (1992) regarding multilayered stress solutions.

In the monolayered case, the cylinder is assumed to be long with ends that are fixed. The cylinder has an inner radius 'a' and an outer radius 'b' (see Figure 1). The

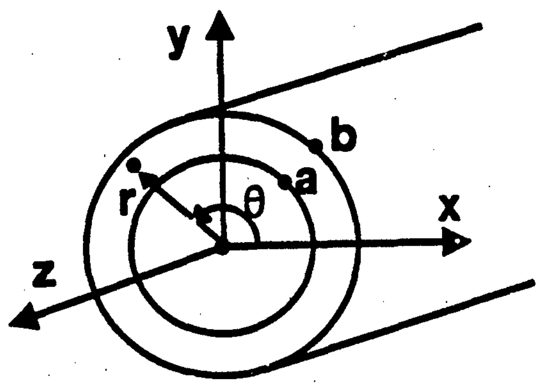


Fig. 1. Monolayered cylinder geometry.

cylinder is subjected to an internal pressure 'p', an external pressure 'q', and a temperature rise of $\Delta T$. An axial force $F_{Z1}$ is necessary to enforce the fixed end constraint generated by the pressures 'p' and 'q', as shown in Figure 2. Similarly, an axial force $F_{Z4}$ is needed to enforce the fixed end constraint for the temperature rise $\Delta T$ (see Figure 3). A cylindrically-orthotropic material is assumed with its principal axes coincident with the cylindrical coordinate system defining the cylinder geometry. In constructing multilayered composite cylinders, layers are most commonly used in + and - fiber wrap angle pairs, where each pair can be viewed as a single orthotropic layer. An orthotropic material is characterized by twelve independent thermo-mechanical material constants consisting of three engineering moduli ($E_1, E_2, E_3$), three shear moduli ($G_{12}, G_{23}, G_{31}$), three Poisson's ratios ($\nu_{12}, \nu_{23}, \nu_{31}$), and three thermal expansion coefficients ($\alpha_1, \alpha_2, \alpha_3$). The numbers 1,2,3 indicate the principal material directions which for the above assumptions correspond to the radial, hoop, and axial directions of the cylinder ($r, \theta, z$). In addition, since the principal material directions correspond to the principal directions of both the cylinder geometry and the applied loadings, shear effects are eliminated, and the number of material constants necessary for the analysis reduces to nine.

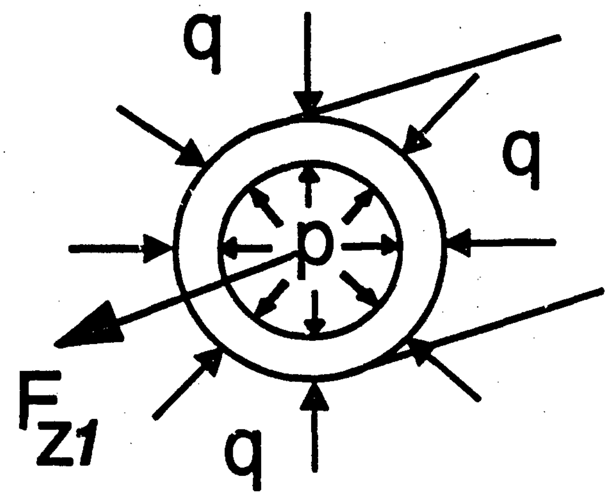


Fig. 2. Internal and external pressure loading of monolayered cylinder under plane-strain boundary conditions.

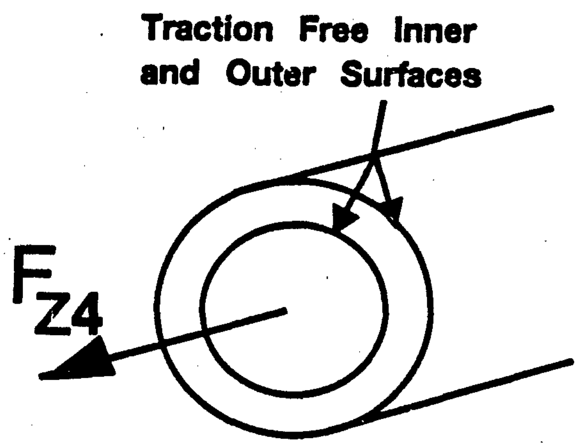


Fig. 3. Uniform temperature change loading for monolayered cylinder under plane-strain boundary conditions.

## MONOLAYERED THERMAL STRESS SOLUTION

The equations describing the thermal stresses for an orthotropic cylinder with the fixed end constraint and traction-free inner and outer surfaces are given by Gerstle and Reuter (1975). These equations, in a slightly different form, are given below. A superscript T is used to signify thermal loading.

$$\begin{Bmatrix}\sigma_r^T\\ \sigma_\theta^T\end{Bmatrix} = D\,\Delta T\begin{bmatrix}1 & 1 & -1\\ k & -k & -1\end{bmatrix}\begin{Bmatrix}\delta\left(\frac{r}{a}\right)^{k-1}\\ (1-\delta)\left(\frac{r}{a}\right)^{-k-1}\\ 1\end{Bmatrix} \tag{1}$$

where 'r' is the radial position in the cylinder, and $\Delta T$ is the uniform temperature change of the layer,

$$D = \frac{(\bar{\alpha}_r-\bar{\alpha}_\theta)}{\beta_{\theta\theta}(k^2-1)} \tag{2}$$

$\beta_{ij}$ are elements of the reduced (plane-strain) compliance matrix as given in

$$\epsilon_i = \beta_{ij}\sigma_j+\bar{\alpha}_i\Delta T \qquad i,j=r,\theta \tag{3}$$

and

$$\beta_{ij} = A_{ij} - \frac{A_{iz}A_{zj}}{A_{zz}} \qquad i,j=r,\theta \tag{4}$$

and $A_{ij}$ are the elements of the compliance matrix for an orthotropic material, as given in the equation relating strain to stress

$$\epsilon_i = A_{ij}\sigma_j+\alpha_i\Delta T \qquad i,j=r,\theta,z \tag{5}$$

and

$$\bar{\alpha}_r = \alpha_r-\alpha_z\frac{A_{rz}}{A_{zz}} \tag{6}$$

$$\bar{\alpha}_\theta = \alpha_\theta-\alpha_z\frac{A_{\theta z}}{A_{zz}} \tag{7}$$

$$k = \sqrt{\frac{\beta_{rr}}{\beta_{\theta\theta}}} \tag{8}$$

$$\delta = \frac{C_o^{-k+1}-1}{C_o^{-2k}-1} \tag{9}$$

$$C_o = \frac{a}{b} \tag{10}$$

In addition, an axial stress is produced as a result of the fixed end constraint and can be obtained by simplifying Eq. (5) for $\epsilon_z = 0$,

$$\sigma_z^T = -\frac{A_{rz}}{A_{zz}}\sigma_r^T - \frac{A_{\theta z}}{A_{zz}}\sigma_\theta^T - \frac{\alpha_z}{A_{zz}}\Delta T \tag{11}$$

The axial force $F_{Z4}$ can be found by integrating the axial stress over the end area

$$F_{Z4} = 2\pi\int_a^b \sigma_z^T r\,dr \tag{12}$$

producing

$$F_{Z4} = -\frac{2\pi D\Delta T}{A_{zz}}[I_1(A_{rz}+kA_{\theta z})+I_2(A_{rz}-kA_{\theta z})+I_3(\frac{\alpha_z}{D}-A_{rz}-A_{\theta z})] \tag{13}$$

where

$$I_1 = \frac{a^2\delta(C_o^{-k-1}-1)}{(k+1)} \tag{14}$$

$$I_2 = \frac{a^2(1-\delta)(C_o^{k-1}-1)}{(1-k)} \tag{15}$$

and

$$I_3 = \frac{b^2-a^2}{2} \tag{16}$$

Later, it will prove necessary to use hoop stress and hoop strain values at the inner and outer surfaces of a layer to construct the multilayered solution. Since the formulation for the monolayered thermal stress solution assumed traction-free inner and outer surfaces, the radial stresses on these surfaces must be zero. Evaluating the hoop stress at these two locations gives

$$\sigma_{\theta,a}^T = D(2k\delta-k-1)\Delta T \tag{17}$$

$$\sigma^T_{\theta,b} = D[k\delta C_o^{1-k} - k(1-\delta)C_o^{k+1} - 1]\Delta T \tag{18}$$

Likewise, the hoop strain at these surfaces is given by

$$\epsilon^T_{\theta,a} = \beta_{22}\sigma^T_{\theta,a} + \bar{\alpha}_\theta \Delta T \tag{19}$$

which simplifies to

$$\epsilon^T_{\theta,a} = \hat{\alpha}_a \Delta T \tag{20}$$

where

$$\hat{\alpha}_a = \beta_{22}D(2k\delta - k - 1) + \bar{\alpha}_\theta \tag{21}$$

Similarly,

$$\epsilon^T_{\theta,b} = \beta_{22}\sigma_{\theta,b} + \bar{\alpha}_\theta \Delta T \tag{22}$$

which simplifies to

$$\epsilon^T_{\theta,b} = \hat{\alpha}_b \Delta T \tag{23}$$

where

$$\hat{\alpha}_b = \beta_{22}D[k\delta C_o^{1-k} - k(1-\delta)C_o^{k+1} - 1] + \bar{\alpha}_\theta \tag{24}$$

## PLANE-STRAIN MONOLAYERED SOLUTION FOR INTERNAL AND EXTERNAL PRESSURES

The equations describing the stress distribution for an orthotropic cylinder with fixed ends and loadings of internal and external pressures are given by Lekhnitskii (1963). The form of these equations, given below, is that suggested by O'Hara (1987). The superscript pq is used to signify the solution for pressure loading

$$\sigma_r^{pq} = \left[\frac{pC_o^{k+1} - q}{(1-C_o^{2k})}\right]\left(\frac{r}{b}\right)^{k-1} + \left[\frac{qC_o^{k-1} - p}{1-C_o^{2k}}\right]C_o^{k+1}\left(\frac{b}{r}\right)^{k+1} \tag{25}$$

$$\sigma_\theta^{pq} = \left[\frac{pC_o^{k+1} - q}{(1-C_o^{2k})}\right]k\left(\frac{r}{b}\right)^{k-1} - \left[\frac{qC_o^{k-1} - p}{1-C_o^{2k}}\right]kC_o^{k+1}\left(\frac{b}{r}\right)^{k+1} \tag{26}$$

$$\sigma_z^{pq} = -\frac{A_{rz}}{A_{zz}}\sigma_r^{pq} - \frac{A_{\theta z}}{A_{zz}}\sigma_\theta^{pq} \tag{27}$$

Similar to what was done to find $F_{Z\theta}$, $F_{Z1}$ can be found by integrating the axial stress over the end area to produce

$$F_{zl} = \frac{2\pi}{A_{33}(1-C_o^{2k})}\left[b^2(q-pC_o^{k+1})(1-C_o^{k+1})\frac{A_{13}+kA_{23}}{1+k} + a^2(qC_o^{k-1}-p)(1-C_o^{k-1})\frac{A_{13}-kA_{23}}{1-k}\right] \tag{28}$$

Again, it will be necessary to have stresses evaluated at the inner and outer surfaces for purposes of constructing the multilayered solution. The radial and hoop stress at these two locations are given below

$$\sigma^{pq}_{r,a} = -p \tag{29}$$

$$\sigma^{pq}_{\theta,a} = TAP \cdot p + TAQ \cdot q \tag{30}$$

where

$$TAP = \frac{k(1+C_o^{2k})}{(1-C_o^{2k})}, \quad TAQ = \frac{-2kC_o^{k-1}}{(1-C_o^{2k})} \tag{31a,b}$$

and

$$\sigma^{pq}_{r,b} = -q \tag{32}$$

$$\sigma^{pq}_{\theta,b} = TBP \cdot p + TBQ \cdot q \tag{33}$$

where

$$TBP = -TAQ \cdot C_o^2, \quad TBQ = -TAP \tag{34a,b}$$

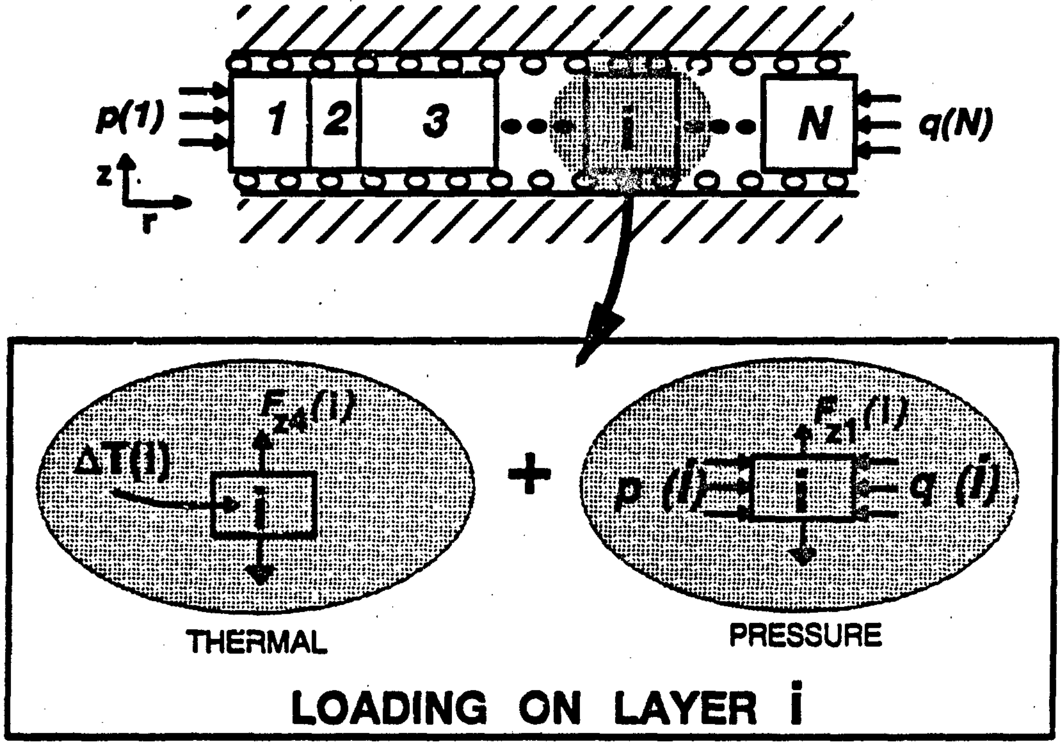


Fig. 4. MSOL4 loading.

**PLANE-STRAIN MULTILAYERED THERMAL STRESS SOLUTION - MSOL4**

As was mentioned earlier, the multilayered thermal stress solution requires both the monolayered thermal stress solution and the monolayered stress solution for loadings of internal and external pressures. This is because as an individual layer shrinks or expands due to its temperature change, it will either push or pull on adjacent layers. This pushing or pulling gives rise to interface pressures at the boundaries of adjacent layers. Figure 4 shows the loading for the MSOL4 solution. In order for continuity of the multilayered cylinder to be maintained for a given temperature distribution, the hoop strain of two layers at their interface must be equal. The equation defining the hoop strain equivalence at the interface of layer i and i+1 is given below

$$\epsilon^{T}_{\theta,b}(i) + \epsilon^{pr}_{\theta,b}(i) = \epsilon^{T}_{\theta,a}(i+1) + \epsilon^{pr}_{\theta,a}(i+1) \qquad (35)$$

By substituting the appropriate stress components from the two monolayered solutions outlined above, we arrive at the following equation:

$$\hat{a}_b(i)\Delta T(i) - \hat{a}_a(i+1)\Delta T(i+1) + G_{i1}\, q(i-1) + G_{i2}\, q(i) + G_{i3}\, q(i+1) = 0 \qquad (36)$$

where

$$G_{i1} = -C^2_a(i)\cdot\beta_{22}(i)\cdot TAQ(i) \qquad (37)$$

$$G_{i2} = -[\beta_{12}(i) - \beta_{12}(i+1) + \beta_{22}(i)\cdot TAP(i) + \beta_{22}(i+1)\cdot TAP(i+1)] \qquad (38)$$

and

$$G_{i3} = -\beta_{22}(i+1)\cdot TAQ(i+1) \qquad (39)$$

For each two-layer interface there is one equation defining the hoop strain equivalence condition. For a cylinder with 'N' orthotropic layers, there are N-1 equations and N-1 unknowns. Setting up these equations in matrix form and noting that p(1) and q(N) are the prescribed internal and external pressures of the overall cylinder results in

$$\begin{bmatrix} G_{12} & G_{13} & & & & & \\ G_{21} & G_{22} & G_{23} & & 0 & & \\ & G_{31} & G_{32} & G_{33} & & & \\ & & - & - & - & & \\ & & & - & - & - & \\ & & & & \cdot & & \\ & 0 & & & \cdot & & \\ & & & & G_{N-2,1} & G_{N-2,2} & G_{N-2,3} \\ & & & & & G_{N-1,1} & G_{N-1,2} \end{bmatrix} \begin{bmatrix} q(1) \\ q(2) \\ q(3) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ q(N-2) \\ q(N-1) \end{bmatrix} = \begin{bmatrix} -G_{11} \cdot p(1) - \Psi(1) \\ -\Psi(2) \\ -\Psi(3) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -\Psi(N-2) \\ -G_{N-1,3} \cdot q(N) - \Psi(N-1) \end{bmatrix} \tag{40}$$

where

$$\Psi(i) = \dot{\alpha}_b(i)\Delta T(i) - \dot{\alpha}_a(i+1)\Delta T(i+1) \tag{41}$$

In a condensed form, the matrix equation above can be written as

$$[G][Q] = [\dot{R}] \tag{42}$$

It should be mentioned that p(1) and q(N) should be set to zero if the objective is to find the multilayered thermal stress solution for traction-free inner and outer surfaces.

The system of equations given above can be easily solved to determine the interface pressure vector [Q]. By recalling that q(i-1) = p(i), we can now calculate the stress distribution in layer i by using ΔT, p(i), and q(i) as input into the two monolayered stress solutions discussed earlier.

Each layer of the cylinder has two axial forces, $F_{Z1}$ and $F_{Z4}$, that make up the total force required to enforce the fixed end conditions. By summing each of these forces for the entire cylinder, we can determine the total axial force ($F_{Z1t}$) from the pressure loads and the total axial force ($F_{Z4t}$) from the thermal loading, as given below

$$F_{Z1t} = \sum_{i=1}^{N} F_{Z1}(i) \tag{43}$$

$$F_{Z4t} = \sum_{i=1}^{N} F_{Z4}(i) \tag{44}$$

## THE MULTILAYERED THERMAL STRESS SOLUTION FOR GENERALIZED PLANE-STRAIN BOUNDARY CONDITIONS - MSOLT

In the multilayered plane-strain thermal solution (MSOL4) discussed in the previous section, the ends of the cylinder are fixed and the axial strain is zero for each layer. Generalized plane-strain boundary conditions address the case in which each layer of the cylinder has equivalent but, in general, nonzero axial strain. Both the fixed ends and free ends type of boundary conditions would be special cases of the generalized plane-strain class of problems. Axial loading can also be modeled when this type of boundary condition is made part of the solutions formulation. By combining MSOL4 with a nonthermal generalized plane-strain solution (MSOLPQF) by Witherell (1992), the multilayered thermal stress solution for generalized plane-strain boundary conditions (MSOLT) can be easily constructed.

The generalized plane-strain multilayered stress solution (MSOLPQF) is for loadings of internal pressure, external pressure, and axial force. This solution is constructed from three separate multilayered solutions, MSOL1, MSOL2, and MSOL3. The MSOLT solution is very simply obtained from the MSOLPQF solution by substituting MSOL4 for MSOL1. Making this substitution also requires adding one more term ($F_{Z4t}$) to the FZN constant used in the MSOLPQF solution. The new equation for FZN is given as

$$FZN = F_{net} - F_{Z1t} - F_{Z4t} \tag{45}$$

## RESULTS

The multilayered thermal stress solution MSOLT was implemented in a FORTRAN program and verified with a finite element solution. In comparing theoretical predictions of maximum absolute values of radial, hoop, and axial stress with those produced using finite elements, a difference of less than 0.4 percent was found. This close agreement is to be expected, since the finite element solution will approach the MSOLT solution as the element size approaches zero.

An example of a solution for a cylinder with an internal radius of 1 inch (2.54 cm), external radius of 2 inches (5.08 cm), with 50 orthotropic layers is shown in Figures 5, 6, and 7. The cylinder is under generalized plane-strain boundary conditions with a net axial force of zero. The layup consists of alternating hoop and axial layers of a graphite/epoxy composite. An IM6 graphite fiber was used, and the composite had a 60 percent fiber volume ratio. The laminate material properties were generated using an average of four micromechanics models and are shown in Table 1.

The cylinder is subjected to a linear radial temperature distribution starting at 50°F at the inner layer, decreasing to 1°F at the outer layer. Also, remember that the temperature within a layer is uniform. Figures 5 and 6 show the thermal stress distribution resulting from this temperature gradient.

**TABLE 1. LAYER MATERIAL PROPERTIES FOR IM6/EPOXY 60 PERCENT FIBER-VOLUME RATIO**

| Fiber Direction | (Mpsi) | | | | | | (10$^{-6}$/°F) | | |
|---|---|---|---|---|---|---|---|---|---|
| | $E_r$ | $E_\theta$ | $E_z$ | $\upsilon_{r\theta}$ | $\upsilon_{\theta z}$ | $\upsilon_{zr}$ | $\alpha_r$ | $\alpha_\theta$ | $\alpha_z$ |
| Hoop | 1.274 | 25.40 | 1.274 | 0.0157 | 0.3127 | 0.3956 | 0.2304 | -0.0263 | 0.2304 |
| Axial | 1.274 | 1.274 | 25.40 | 0.3956 | 0.0157 | 0.3127 | 0.2304 | 0.2304 | -0.0263 |

1 Mpsi = 6.89 GPa
$1^{\circ}F^{1}$ = $9/5^{\circ}C^{1}$

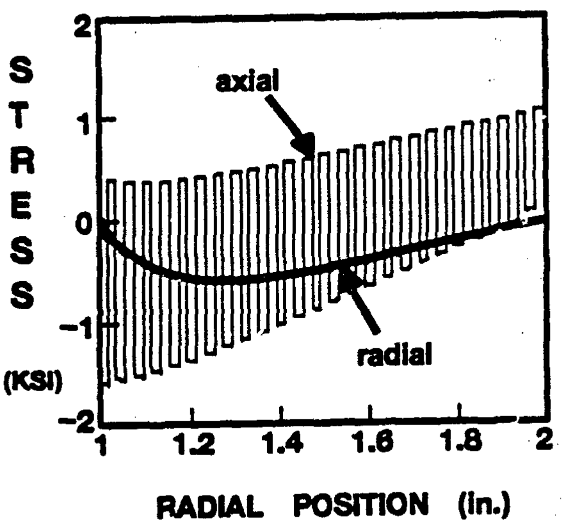


Fig. 5. Axial and radial thermal stress distribution (1 Ksi = 6.89 MPa, 1 in. = 2.54 cm).

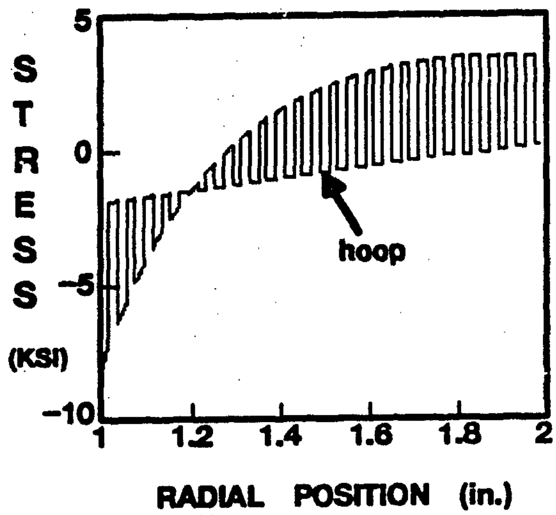


Fig. 6. Hoop thermal stress distribution (1 Ksi = 6.89 MPa, 1 in. = 2.54 cm).

Because the inner and outer surfaces are traction-free, the radial stress at these locations is zero. The large fluctuations in hoop and axial stresses occur at axial-hoop layer boundaries. The thermal strain distribution is shown in Figure 7. The radial strain curve has some jaggedness to it because of jumps in temperature from layer to layer.

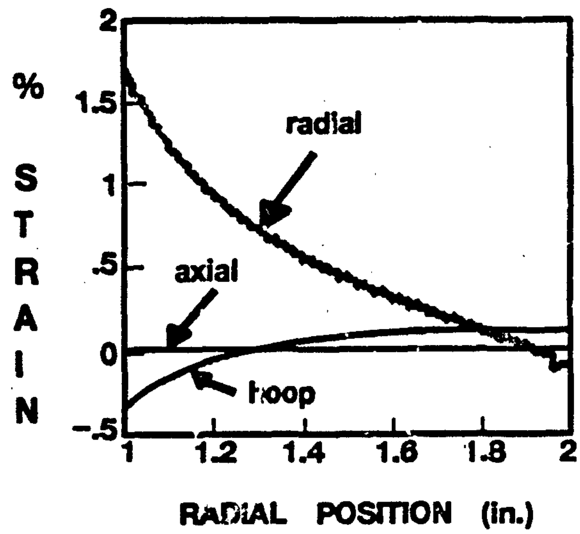


Fig. 7. Axial, radial, and hoop thermal strain distribution (1 in. = 2.54 cm).

## CONCLUSIONS

Closed-form elastic stress solutions for monolayered cylinders under thermal and pressure loading were used to construct the plane-strain multilayered thermal stress solution. In addition, it was shown that this solution can be simply extended to the case of generalized plane-strain boundary conditions. The solution has been compared with finite element results with excellent agreement. Finally, the multilayered thermal stress solution has been successfully integrated with a finite difference heat transfer solution to predict transient stress distributions in multilayered cannon.

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